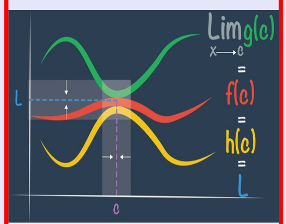
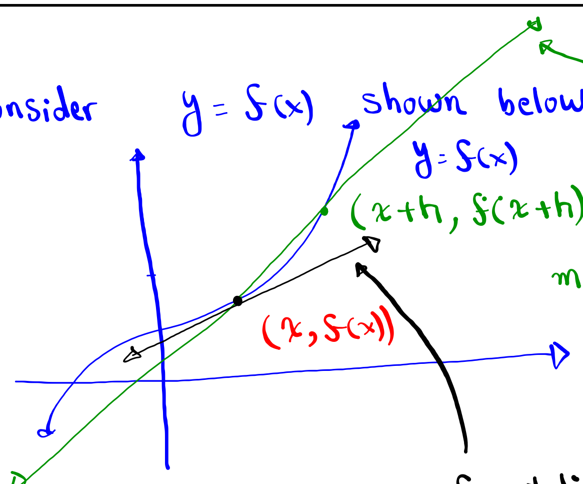
Math 261

Spring 2021

Lecture 10



Consider $y = f(x)$ shown below



$$m_{\text{secant line}} = \frac{f(x+h) - f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$$

As $h \to 0$ Secant line becomes tangent line at the Point $(x, f(x))$

$$\text{Slope}_{\text{tan. line}} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

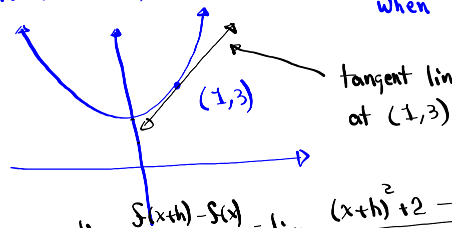
if the limit exist.

$$\boxed{m_{\text{tan. line}} = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}}$$

Given $f(x) = x^2 + 2$

when $x=1$

$f(1)=3$



$$m_{\text{tan. line}} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2+2-(x^2+2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2\cancel{+2}\cancel{-x^2}\cancel{-2}}{h}$$

$$= \lim_{h\to 0} \frac{2xh+h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(2x+h)}{h} = \lim_{h\to 0}(2x+h)$$

$$= 2x+0 = \boxed{2x}$$

Slope of the tangent line at (1,3) is

$m = 2(1) = 2$

Equation of the tangent line

$y - y_1 = m(x - x_1)$

$y - 3 = 2(x-1)$

$\Rightarrow \boxed{y = 2x+1}$

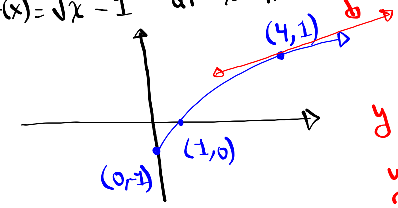
Find equation of tan. line to the graph of

$f(x) = \sqrt{x} - 1$ at $x=4$.

Find $f(4)$

$f(4) = \sqrt{4} - 1$

$= 1$



$y - y_1 = m(x - x_1)$

$y - 1 = \frac{1}{4}(x-4)$

$\boxed{y = \frac{1}{4}x}$

$$m_{\text{tan. line}} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h}-1-(\sqrt{x}-1)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$= \lim_{h\to 0} \frac{\cancel{x}+\overset{1}{\cancel{h}}-\cancel{x}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}$$

at tan. Point (4,1)

$m = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

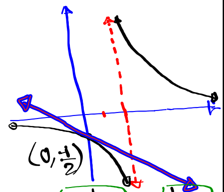
$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{\sqrt{x+0}+\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$$

Find equation of the tangent line to the graph of $f(x)=\frac{1}{x-2}$ at the point where $x=0$.

Domain $(-\infty,2)\cup(2,\infty)$

V.A. at $x=2$

$f(0)=\frac{-1}{2}$



$y-y_1=m(x-x_1)$

$y-\frac{-1}{2}=\quad(x-0)$

$$m_{\text{tan. line}}=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to0}\frac{\boxed{\frac{1}{x+h-2}}-\boxed{\frac{1}{x-2}}}{h}$$

LCD $=(x+h-2)(x-2)$

$$=\lim_{h\to0}\frac{x-2-(x+h-2)}{h(x+h-2)(x-2)}=\lim_{h\to0}\frac{-\cancel{h}^{\,-1}}{\cancel{h}(x+h-2)(x-2)}$$

$$=\lim_{h\to0}\frac{-1}{(x+h-2)(x-2)}=\frac{-1}{(x+0-2)(x-2)}=\frac{-1}{(x-2)^2}$$

at $(0,\frac{-1}{2})$

$$m=\frac{-1}{(0-2)^2}=\frac{-1}{4}$$

$y-y_1=m(x-x_1)$

$y-\frac{-1}{2}=\frac{-1}{4}(x-0)$

$$\boxed{y=\frac{-1}{4}x-\frac{1}{2}}$$

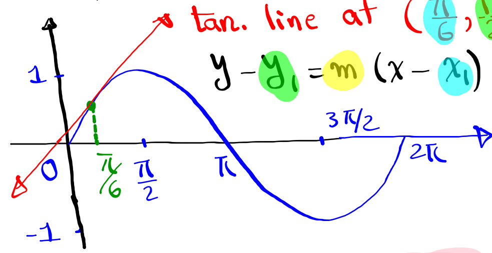
Find the equation of the tangent line to the graph of $f(x)=\sin x$ at the point with $x=\frac{\pi}{6}$.

tan. line at $(\frac{\pi}{6},\frac{1}{2})$

$y-y_1=m(x-x_1)$

$f(\frac{\pi}{6})=$

$\sin\frac{\pi}{6}=\frac{1}{2}$



$$m_{\text{tan. line}}=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to0}\frac{\sin(x+h)-\sin x}{h}$$

$$=\lim_{h\to0}\frac{\sin x\cos h+\cos x\sin h-\sin x}{h}$$

$$=\lim_{h\to0}\frac{\sin x(\cos h-1)+\cos x\sin h}{h}$$

Recall

$$\lim_{h\to0}\frac{\sin h}{h}=1$$

$$\lim_{h\to0}\frac{1-\cos h}{h}=0$$

$$= \lim_{h \to 0} \frac{\sin x(\cos h - 1) + \cos x \sin h}{h}$$

$$= \lim_{h \to 0} \left[\frac{\sin x(\cos h - 1)}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h \to 0} \frac{\sin x(\cos h - 1)}{h} + \lim_{h \to 0} \frac{\cos x \sin h}{h}$$

$$= \sin x \underbrace{\lim_{h \to 0} \frac{\cos h - 1}{h}}_{\to 0} + \cos x \underbrace{\lim_{h \to 0} \frac{\sin h}{h}}_{\to 1}$$

$m = \cos x$

but we want at $\left(\frac{\pi}{6}, \frac{1}{2}\right)$

$$m_{\text{tan. line}} = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$y - \frac{1}{2} = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$$

$$y = \frac{\sqrt{3}}{2} x + \frac{1}{2} - \frac{\pi\sqrt{3}}{12}$$

Class QZ 4

For $\varepsilon > 0$, find $\delta > 0$ such that

$$\lim_{x \to 2} (x^2 + 3x - 2) = 8.$$